Heterogeneous Intermediary Asset Pricing – A Comment*

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Abstract

Economic models in the literature of intermediary asset pricing usually are presented in a calibration setting and used as theoretical base for the construction of factor models that are supposed to explain cross-sectional variation in returns. One of these is the Kargar (2021) model. While the factor model arising from the DSGE framework performs rather well, there are issues with the DSGE model that should be addressed.

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1 Introduction

Asset pricing has turned to various hypotheses that should explain asset returns and solve the equity premium puzzle (Mehra & Prescott, 1985). One of these hypotheses focuses on the preferences of financial intermediaries as marginal pricers in various asset markets. Research develops in many different directions within this literature strand with some authors identifying intermediary-based risk factors to price the cross-section of asset returns (see He et al. (2017) and Adrian et al. (2014) for the most influential studies), others focusing on characterizing an intermediary stochastic discount factor (Ma, 2023) and again different researchers establishing DSGE models supposed to explain the dynamics of economies with financial intermediaries (He & Krishnamurthy, 2012, 2013).

In 2021, a paper under the name "Heterogeneous Intermediary Asset Pricing" by Mahyar Kargar, published in the Journal of Financial Economics, set out to combine two of these fields with the establishment of a DSGE model and the identification of a new intermediary risk factor. This new risk factor offers additional explanatory power over the intermediary factor proposed by He et al. (2017) by accounting for the heterogeneity of the financial sector.

This paper is the topic of this comment. While the performance of the intermediary risk factor proposed by Kargar is highly interesting and appealing from a viewpoint of theoretical considerations and past empirical results, I want to focus my attention to the presentation of the DSGE model that the author uses in order to motivate the intermediary risk factor in the first place as there are some issues of varying significance that hinder the replication of results. There are also some contradictions and other problems that should be addressed in the interest of other interested researchers that may want to turn to the analysis of this model.

This short article is therefore structured as follows. Section 2 presents the DSGE framework by Kargar, while Section 3 discusses the issues, errors and problems of the paper. Section 4 concludes this comment.

2 The Kargar Model

2.1 Model Framework

Kargar (2021) presents a DSGE model with three agents that is set in continuous time. The model abstracts from the production side of the economy and instead assumes an aggregate dividend process that serves as endowment for the investors. This process evolves according to

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D dZ_t, \tag{2.1}$$

where μ_D and σ_D are constants and dZ_t follows a standard Brownian motion. It captures permanent shocks to aggregate dividends and serves as the only source of uncertainty within the model. At no other point are additional shocks introduced into the environment such that dZ_t is the sole driver of all model dynamics. The endowment growth rate $g_{D_t} = \mu_D - \sigma_D^2/2$ is assumed to be positive.

The agents endowed with this aggregate dividend process are placed within a continuum of agents i with mass 1 that is split into three population groups. All groups share the same functional form of Epstein-Zin preferences that allow the disentanglement of the Elasticity of Intertemporal Substitution (EIS, ψ_i) and the relative risk aversion (γ_i) (Epstein & Zin, 1989). The lifetime utility of agent i is defined as

$$U_{i,t} = \mathbb{E}_t \left[\int_t^\infty \underbrace{\left(\frac{1-\gamma_i}{1-1/\psi_i}\right) U_{i,s} \left[\left(\frac{C_{i,s}}{[(1-\gamma_i)U_{i,s}]^{1/(1-\gamma_i)}} \right)^{1-1/\psi_i} - (\rho+\kappa) \right]}_{f_i(C_{i,s},U_{i,s})} ds \right]. \tag{2.2}$$

The time-preference rate ρ and the exogenous mortality risk κ are shared parameters for all agents.

A share u of the agents takes on the role of an aggressive financial intermediary that can be understood as a stand-in for broker dealers or hedge funds. These agents have the lowest risk aversion in the economy and therefore strive for the highest possible leverage in the risky asset. Crucially, however, these agents face a time-varying and occasionally binding margin constraint that may restrict their ability to take on leverage during times of financial distress or heightened volatility. In the model notation, these are denoted as agent A. A further population share v represents commercial bank holdings, another type of financial intermediary. However, this type exhibits a slightly more risk averse behavior and thus is not as highly leveraged as the aggressive intermediary. However, since they do not face the same margin constraints, these intermediaries are supposed step in to clear the asset market when the A-types are restricted in their choices. The remaining share of the population $(1-u-v)$ represents the households. The households are the most risk averse players in the economy and serve as lenders for the intermediaries who are assumed to be unable to issue equity and instead finance their holdings of the risky asset through debt. This results in a risk aversion parameter structure of $\gamma_A < \gamma_B < \gamma_C$.

Financial markets offer access to both an instantaneous risk-less bond in zero net supply that is traded among the agents and a risky asset serving as a claim on the endowment level D_t, the return of which evolves according to

$$dR_t = \frac{dP_t + D_t dt}{P_t} \equiv \mu_t dt + \sigma_t dZ_t. \tag{2.3}$$

Here, P_t is the price of the risky asset, μ_t represents its expected return and σ_t stands for the asset's volatility. The price of the consumption good is normalized to one as it is used as numéraire such that the price of the risky asset is expressed in terms of consumption. The risky asset's dividend-price ratio can be expressed as $F_t = D_t/P_t$.

The dynamic budget constraint holds for all agents and takes on the form

$$\frac{dW_{i,t}}{W_{i,t}} = \left(r_t + w^i_{s,t}(\mu_t - r_t) - c_{i,t}\right) dt + w^i_{s,t}\sigma_t dZ_t, \tag{2.4}$$

where $W_{i,t}$ denotes the wealth level of agent i at time t. The initial condition $W_{i,0} > 0$ is assumed to hold for all agents. Agents earn the interest rate r_t on their risk-less bonds and an excess return of $\mu_t - r_t$ on their holdings of the risky asset financed through selling bonds. The portfolio weight invested in the risky asset is denoted as $w^i_{s,t}$ and also enters through the aggregate dividend shocks that affect the asset's return. A weight of $w^i_{s,t} > 1$ indicates a leveraged position in the risky asset, something that will in practice be exclusive to the two intermediaries A and B. The consumption-to-wealth ratio is referenced as $c_{i,t} = \frac{C_{i,t}}{W_{i,t}}$.

As previously mentioned, the aggressive intermediaries A face a state-dependent margin constraint θ_t that limits their leverage. This restriction depends on the return volatility σ_t and takes the form

$$w^A_{s,t} \leq \theta_t = \frac{1}{\alpha\sigma_t}. \tag{2.5}$$

The parameter α determines the overall tightness of the constraint, but the exact level is endogenously determined within the model through the evolution of the return process.

Within each faction, all individuals are homogeneous such that the respective optimization problem can be analyzed from the perspective of a representative agent. They decide on their consumption-to-wealth ratio path $c_{i,t} > 0$ and their portfolio weights $w^i_{s,t}$ that maximize their lifetime utility defined in (2.2) under their dynamic budget constraint (2.4), a solvency constraint $W_{i,t} > 0$ and, in case of the A-type, the financial constraint (2.5). Formally, this becomes

$$V_{i,t} = \max_{c_{i,t}>0, w^i_{s,t}} U_{i,t} \quad \text{s.t.} \quad (2.4), (2.5), W_{i,t} \geq 0. \tag{2.6}$$

Kargar (2021, p.513) defines a competitive equilibrium as the set of aggregate stochastic processes adapted to the information set generated by Z_t: the price of the risky asset P_t representing a claim on future endowment and the risk-free interest rate r_t. Additionally, it contains agents' individual net worth W_i, consumption C_i, and the risky asset's portfolio weights w_s^i such that

1. Given the stochastic processes (P_t, r_t), the agents' individual choices $(C_{i,t}, w_{s,t}^i)$ solve the previously defined optimization problem in Equation (2.6).

2. The goods market (2.7), the stock market (2.8) and the bond market (2.9) all clear. Note that bond market clearing implies $W_{A,t} + W_{B,t} + W_{C,t} = W_t = P_t$.

$$C_{A,t} + C_{B,t} + C_{C,t} = D_t \tag{2.7}$$

$$w_{s,t}^A W_{A,t} + w_{s,t}^B W_{B,t} + w_{s,t}^C W_{C,t} = P_t \tag{2.8}$$

$$(1 - w_{s,t}^A)W_{A,t} + (1 - w_{s,t}^B)W_{B,t} + (1 - w_{s,t}^C)W_{C,t} = 0 \tag{2.9}$$

Due to the homothetic Epstein-Zin preferences implying optimal consumption paths that are linear in the agents' wealth, the entire model and its objects can be characterized by two state variables which represent the wealth share of the intermediary sector in the entire economy (x_t) and the wealth share of the aggressive broker-dealers/hedge funds of the A-type within the financial sector (y_t). Consequently, they are defined as

$$x_t = \frac{W_{A,t} + W_{B,t}}{W_t} \quad \text{and} \quad y_t = \frac{W_{A,t}}{W_{A,t} + W_{B,t}}, \quad x, y \in [0, 1]. \tag{2.10}$$

This sets the model apart from previous intermediary asset pricing models such as the one presented by He and Krishnamurthy (2013) that only assume one type of financial intermediary and, therefore, require only one state variable.

For the characterization of the dynamic system in the state space $(x, y) \in [0, 1] \times [0, 1]$, Kargar (2021) then derives laws of motion for these state variables and the objects that ultimately depend on them. They evolve according to Itô processes that exhibit a mean-reverting characteristic[1]:

$$\begin{aligned}
dx = \kappa(\bar{x} - x)dt + \mu_x dt + \sigma_x dZ \;\Rightarrow\; \mu_x =& x\Big[\left(y w_s^A + (1-y) w_s^B - 1\right)\left(\mu - r - \sigma^2\right) \\
&- (y c_A + (1-y) c_B) + F\Big] \\
\sigma_x =& x\left(y w_s^A + (1-y) w_s^B - 1\right)\sigma \\
dy = \kappa(\bar{y} - y)dt + \mu_y dt + \sigma_y dZ \;\Rightarrow\; \mu_y =& y(1-y)\big[(w_s^A - w_s^B)(\mu - r) - c_A + c_B \\
&- \left[y w_s^A + (1-y) w_s^B\right](w_s^A - w_s^B)\sigma^2\big] \\
\sigma_y =& y(1-y)\left(w_s^A - w_s^B\right)\sigma
\end{aligned} \tag{2.11}$$

[1] Time indices omitted for brevity.

Based on these definitions, the return process in Equation (2.3) can be rewritten as a function of x_t and y_t. Furthermore, the optimization problem mentioned in Equation (2.6) can be written recursively in the form of Hamilton-Jacobi-Bellman (HJB) equations. They take the form

$$0 = \max_{c_i, w_s^i} h_i(c_i W_i, V_i(W_i, x, y))dt + \mathbb{E}_t\Big[dV_i(W_i, x, y)\Big]. \tag{2.12}$$

The author's conjecture for the value function of agent i, $V_i(\cdot)$, which has the power form due to the homothetic Epstein-Zin preferences, reads:

$$V_i(W_i, x, y) = \frac{W_i^{1-\gamma_i}}{1-\gamma_i} J_i(x, y)^{\frac{1-\gamma_i}{1-\psi}}, \tag{2.13}$$

where $J_i(x, y)$ is some unknown agent-specific function $\mathbb{R}^2 \mapsto \mathbb{R}$. Solving the recursive optimization problem yields the following optimal choices for c_i and w_s^i:

$$c_i^* = J_i(x, y), \tag{2.14}$$

$$w_s^{i,*} = \frac{1}{\psi}\left[\frac{\mu - r}{\sigma^2} + \frac{1-\gamma_i}{1-\psi}\left(\frac{J_{i,x}}{J_i} x \frac{\sigma_x}{\sigma} + \frac{J_{i,y}}{J_i} y(1-y)\frac{\sigma_y}{\sigma}\right)\right]. \tag{2.15}$$

$J_i(x, y)$ therefore represents agent i's optimal consumption-to-wealth ratio c_i^*. This allows the reformulation of the recursive HJB equations into a system of second-order partial differential equations (PDEs) which are the key to obtain the model solution:

$$\begin{aligned}\rho + \kappa =& \frac{1}{\psi} J_i + \left(1 - \frac{1}{\psi}\right)\left[r + w_s^i(\mu - r) - \frac{\gamma_i}{2}\left(w_s^i\right)^2 \sigma^2\right] - \frac{1}{\psi}\left(\left[\frac{J_{i,x}}{J_i}\left(\kappa(\bar{x} - x) + \mu_x\right)\right.\right. \\ &\left. + \frac{J_{i,y}}{J_i}\left(\kappa(\bar{y} - y) + \mu_y\right)\right] + (1-\gamma_i) w_s^i \sigma\left[\frac{J_{i,x}}{J_i}\sigma_x + \frac{J_{i,y}}{J_i}\sigma_y\right]\Bigg) \\ &- \frac{1}{2\psi}\left[\frac{\psi - \gamma_i}{1-\gamma_i}\left(\frac{J_{i,x}}{J_i}\sigma_x + \frac{J_{i,y}}{J_i}\sigma_y\right)^2 + \frac{J_{i,xx}}{J_i}\sigma_x^2 + \frac{J_{i,yy}}{J_i}\sigma_y^2 + 2\frac{J_{i,xy}}{J_i}\sigma_x\sigma_y\right].\end{aligned} \tag{2.16}$$

Finally, the equilibrium conditions can be reworked into a recursive Markov equilibrium that is defined as a set comprising (policy) functions and laws of motion for the endogenous variables (see Kargar (2021, p.515)). The set of functions encompasses the unknown $J_i(x, y)$, the dividend-price ratio $F(x, y)$ and the real interest rate $r(x, y)$. The policy functions consist of the decision rules for the variables $c_i(x, y)$ and $w_s^i(x, y)$, while the laws of motion for both state variables have been previously defined. This set is chosen such that

1. The functions $J_i(x, y)$ solve the agents' HJB equations defined in Equation (2.12) with $c_i(x, y)$ and $w_s^i(x, y)$ being the corresponding policy functions and the remaining components being taken as given.

2. The goods (2.17) and stock (2.18) markets clear:

$$xyc_A + x(1-y)c_B + (1-x)c_C = F \tag{2.17}$$

$$xyw_s^A + x(1-y)w_s^B + (1-x)w_s^C = 1 \tag{2.18}$$

3. The state variables' laws of motion satisfy (2.11).

Solving the system of PDEs in Equation (2.16) yields the necessary policy functions to compute the equilibrium of the economy, resulting in a global solution for a given parameter choice. The details of the used method will be discussed in the following subsection. Kargar's (2021) further line of arguments hinges on the calibration of the model's preference and endowment parameters, which is presented in Table 1.

	Parameter	Calibrated Values
	Preferences	
ψ_A	EIS of type A	1.5
ψ_B	EIS of type B	1.5
ψ_C	EIS of type C	1.5
γ_A	Risk aversion of type A	2.5
γ_B	Risk aversion of type B	5.5
γ_C	Risk aversion of type C	15
ρ	Rate of time preference	0.001
	Endowment and demography	
μ_D	Endowment drift	0.022
σ_D	Endowment volatility	0.035
κ	Agents' entry/exit rate	0.015
$\bar{u}$	Population share of type A	0.05
$\bar{v}$	Population share of type B	0.07
	Margin constraint	
α	Tightness of dynamic constraint	10

Table 1: Calibrated parameter values as proposed by Kargar (2021).

2.2 Solution Method

The model does not possess an analytical solution. Instead, it needs to be solved by Chebyshev collocation. This projection method is used to approximate functions with a sum of so-called Chebyshev polynomials that can be defined as trigonometric functions:

$$T_n(\omega) = \cos(n \arccos \omega) \tag{2.19}$$

By construction, these polynomials are both defined on and bounded between -1 and 1. They are scaled with agent-specific coefficient vectors $\mathbf{a}^i$ that, in theory, should decrease geometrically for polynomials of increasing order. These $\mathbf{a}^i$ are the parameters that need to be identified in order to solve the Kargar model, as we need to approximate three functions capturing the consumption-to-wealth ratio of the three agents in the economy:

$$\tilde{J}_i(x,y) = \sum_{j=0}^{N} \sum_{k=0}^{N} a^i_{jk} \mathbf{\Psi}_{j,k}(x,y) \qquad \text{for } i = A, B, C, \tag{2.20}$$

where the multivariate tensor basis is $\mathbf{\Psi}_{j,k}(x,y) = T_j(\omega(x))T_k(\omega(y))$ with $T_j(\omega(z))$ representing the Chebyshev polynomial of order j evaluated at $\omega(z) = 2z - 1$. This function represents a transformation from the state space of the state variables x and y, which are both defined on $[0,1]$, to the domain of the Chebyshev polynomials defined on $[-1,1]$. Kargar uses $N = 19$ for a total of 1,200 coefficients. However, a reduction to $N = 8$ yields a greatly improved performance with only 243 necessary coefficients at only marginal accuracy losses.

The polynomials are evaluated on a grid of the roots of the Chebyshev polynomial of order $N+1$, as is common practice (e.g., see Fernández-Villaverde et al. (2016)). With these concrete values for the approximated functions, the HJB equations (2.12) serving as residuals for the optimization that must be set to zero, can be explicitly evaluated. The calculation of these equations relies on a numerical algorithm that is (partly) provided in the paper's internet appendix. The complete version, which also includes a crucial system of equations to determine the correct portfolio weights, is presented in Appendix A[2]. In Kargar's internet appendix, this adjustment is missing, making it impossible for interested researchers to solve the optimisation problem without any further ado.

3 Critique

The missing system of equations in the paper's internet appendix is not the sole issue with the presentation of this model. There are some flaws within the paper that should to be pointed out and explained for other researchers wishing to build on this framework.

[2]We thank Nicole Branger (University of Münster) for providing the updated system of equations.

First of all, while this is a model of heterogeneous agents, one of which is constrained by a margin constraint, there is no actual intermediation mechanic at work in the background. Other papers in the field of intermediary asset pricing presented by authors such as He and Krishnamurthy (2012, 2013) or Baron and Muir (2022) build their reasoning on the prevalence of moral hazard problems that govern the relationship between intermediaries and households that cannot directly invest in the risky asset. In the present model, households are differentiated from intermediaries only through their heightened risk aversion. Apart from this feature, they possess the same capabilities of investing in the risky asset or the risk-free instantaneous bond. The model does not specify how the intermediary channel would actually be operated, how contracts are formed or how the households even require the intermediaries for their investment decisions. These mechanics are instead completely absent. As such, the model does depict an interesting economy of three different agents but not much more from a standpoint of intermediary asset pricing itself.

A second point that is interesting to note is the fact that the author builds on his model to construct a new intermediary risk factor for which he measures empirical counterparts to the state variables x and y. Yet, the calibration of the model makes it impossible to achieve the level of the state variables that is observed in the data. This is a peculiar fact that came to light in the estimation of the model's preference parameters discussed in a separate paper. There, simulation-based estimation strategies using first or second uncentered moments of the state variables failed due to the incompatibility of simulated and observed levels. Given their central role, a calibration closer to actual observed values would have been more intuitive but this aspect of the study is not discussed in the paper at all.

A further point that lacks discussion or proof within the original paper is the origin of the state variables' laws of motion as characterized in Equation (2.11). Using Itô's lemma, it is possible to compute both drifts and diffusions of the state variables as indicated in the paper. However, the mean reversion characteristic introduced by $\kappa(\bar{x} - x)dt$ and $\kappa(\bar{y} - y)dt$, respectively, is an arbitrary introduction to the laws of motion that is not well communicated. These properties may be a desirable outcome for the behavior of the state variable dynamics, but arbitrary changes that do not arise from theory and analytical derivations should be indicated and introduced as such.

As mentioned before, the two agents A and B that are supposed to capture the behavior of broker-dealers and commercial bank holdings, respectively, stand at the center of the model. Due to the margin constraint restricting dealers in the model, the author claims that, in times of distress for the financial sector (i.e. low values of x), agent B's leverage should surpass that of agent A. This diverging response to tightening wealth conditions is supposed to capture a degree of heterogeneity that is observed in the financial sector empirically. There, arguments for the countercyclicality of leverage ratios for commercial banks have been put forth by authors such as Di Tella (2019) or He et al. (2017). Simultaneously, Adrian et al. (2014) lend support to the hypothesis of procyclical behavior of broker-dealers' leverage ratios, such that modeling the financial sector as a homogeneous group may, as a result, incur inaccuracies, which is where the Kargar model comes into play. However, apart from the fact that agent A's leverage is not really behaving in a procyclical fashion in the paper in the first place, there is a much larger issue undermining the discussion: two plots describing the model's aforementioned central features presented by the author contradict each other in rather dramatic fashion. Kargar claims that the parametrization shown in Table 1 produces all relevant figures in the paper. This is a false claim that can rather easily be spotted by a simple analysis. The author presents a proof that shows boundary conditions for the state variable diffusions being equal to zero for the respective state variable approaching either 0 or 1. The point that is missed in this proposition, however, is the fact that while the boundary conditions do hold by construction, no statement is made about the general sign of the diffusions. While σ_x should, in fact, remain positive over the entire state space if agent C acts as a net lender to the other agents, the same cannot be said about its counterpart σ_y. Given Equation (2.11), we know that the diffusion of state variable y is directly linked to the relationship of the leverage of the two financial intermediaries. To recall, we write

$$\sigma_y = y(1-y)(w_s^A - w_s^B)\sigma.$$

Since y is defined on $[0,1]$, the first two factors are strictly non-negative. However, in the scenario of binding margin constraints, for which Kargar claims that his parametrization produces Figure 4 (p.520), we apparently observe the B-type intermediary taking on more leverage than its counterpart A who is restricted by the constraint. In this case, we can easily observe that $(w_s^A - w_s^B) < 0$, such that the third factor is negative. Paired with the return volatility σ being strictly positive in all considered combinations of preference parameters, the state variable diffusion for the wealth share of the aggressive intermediary within the financial sector must therefore become negative whenever the conservative actor in the sector overtakes the aggressive player in leverage, which Kargar claims is happening at the given calibration. However, Figure 6 (p.523) demonstrates strictly non-negative values for the state variable diffusions. I include both graphs in Figures 6 and 7 in this paper's Appendix B for illustration purposes.

This leaves two possibilities: either the parameters do, in fact, generate intersecting leverage dynamics, such that the diffusions in Figure 6 are wrong but Figure 4 remains plausible. Or, the suggested parametrization does not achieve the desired equilibrium characteristics and agent B's leverage never exceeds that of A even when the margin constraints bind. In that case, at least the state variable diffusions would be achievable. My replication at the calibrated parameters in Table 1 shows that the latter version is the case for the present paper. I show the generated leverage dynamics in Figure 1, where the level of agent A's leverage ratio closely coincides with the paper but is never overtaken by its counterpart B. Though, as a result, the corresponding diffusions of the two state variables that are shown in Figure 2 are virtually identical to the strictly non-negative ones depicted by Kargar (2021). The proposed parameter combination does therefore not produce the suggested leverage dynamics that are the centerpiece for the author's line of argument. In fact, no single parameter combination can produce both strictly positive state variable dynamics and intersecting leverage behavior. This is a major issue for the overall argument behind the DSGE model, especially since this property at the calibrated parameters is explicitly claimed by the author. If the model does not produce the desired heterogeneity in the actions of the intermediaries in response to crisis episodes, it does not necessarily lend support to the choice for a new intermediary capital risk factor for cross-sectional asset pricing tests. While the proposed risk factor performs well in asset pricing tests, it appears to be founded on shaky theoretical grounds.

The choice of calibrated parameters corresponds approximately to the model objects contained in Kargar's Figure 2 (p.518) that is included in Appendix B of this paper as Figure 8. Some minor discrepancies extend to this graph as the levels of some objects such as the risk premium or the Sharpe ratio do not exactly coincide with the official results in the paper as shown in Figure 3. These inaccuracies can also be reproduced with code provided by the author himself such that this appears to be a reporting mistake. However, this may be more of a quantitative issue as the general message of the plots remains unchanged regardless of the exact values.

Finally, I still want to raise the point that the model is, in fact, generally capable of recreating the leverage plot shown in the paper, albeit with a different parametrization. I achieve this by reducing agent B's risk aversion γ_B from 5.5 to 3, such that the intermediaries are a lot closer in their preferences than before. This yields a leverage plot that is extremely close to the one shown in the paper but is also associated with a diffusion parameter for y that turns significantly negative in regions of the state space where agent B takes on more leverage than intermediary A. These results are shown in Figures 4 and 5. Since the author stresses the importance of the different

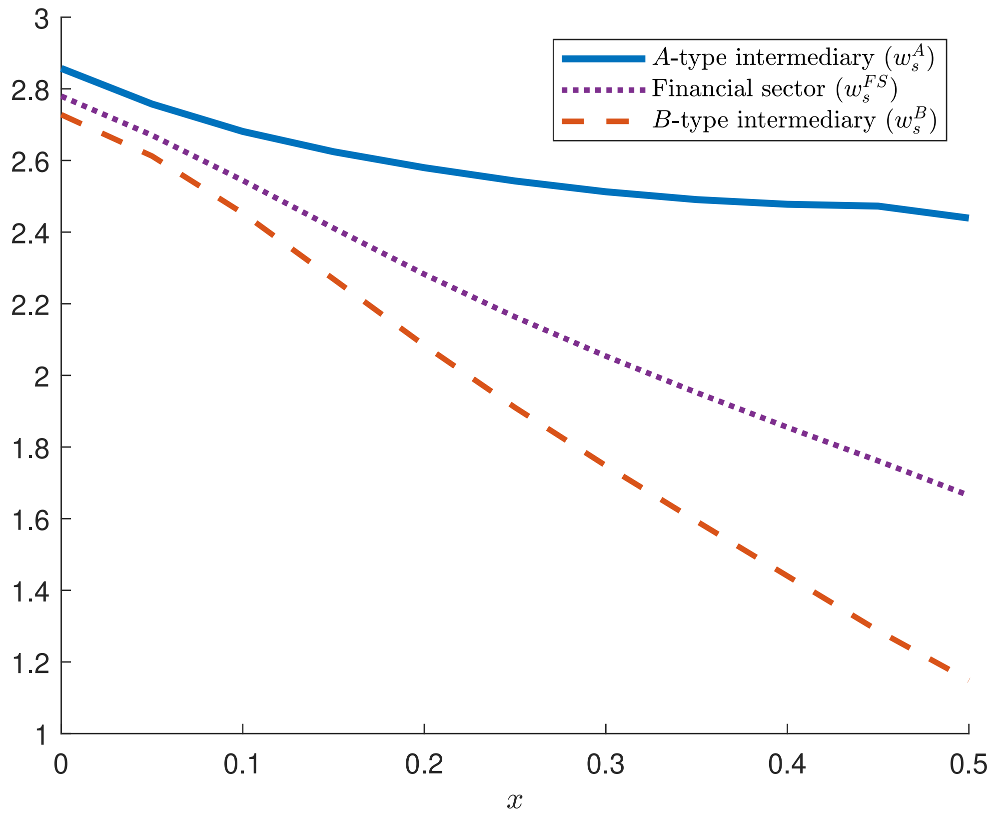

Figure 1: Leverage Plot with Calibrated Parameters

This graph depicts the actual leverage ratios of agents A and B at the calibrated parameter values presented in Table 1. It therefore stands in direct opposition to Kargar's Figure 4 who claims to generate higher leverage ratios for agent B when A is constrained in lower regions of x.

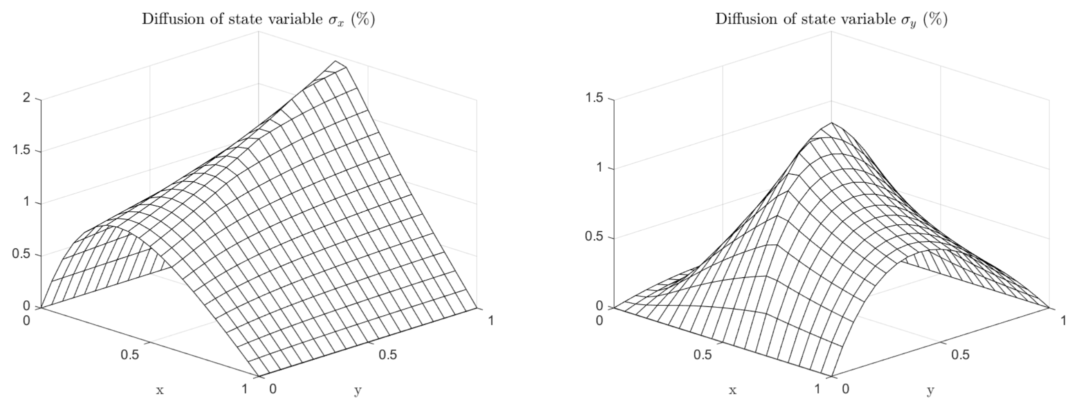

Figure 2: Diffusion Plots with Calibrated Parameters

This figure shows the state variable diffusions σ_x and σ_y at the calibrated parameter values (Table 1). The graphs coincide with the results in the original paper both qualitatively and quantitatively.

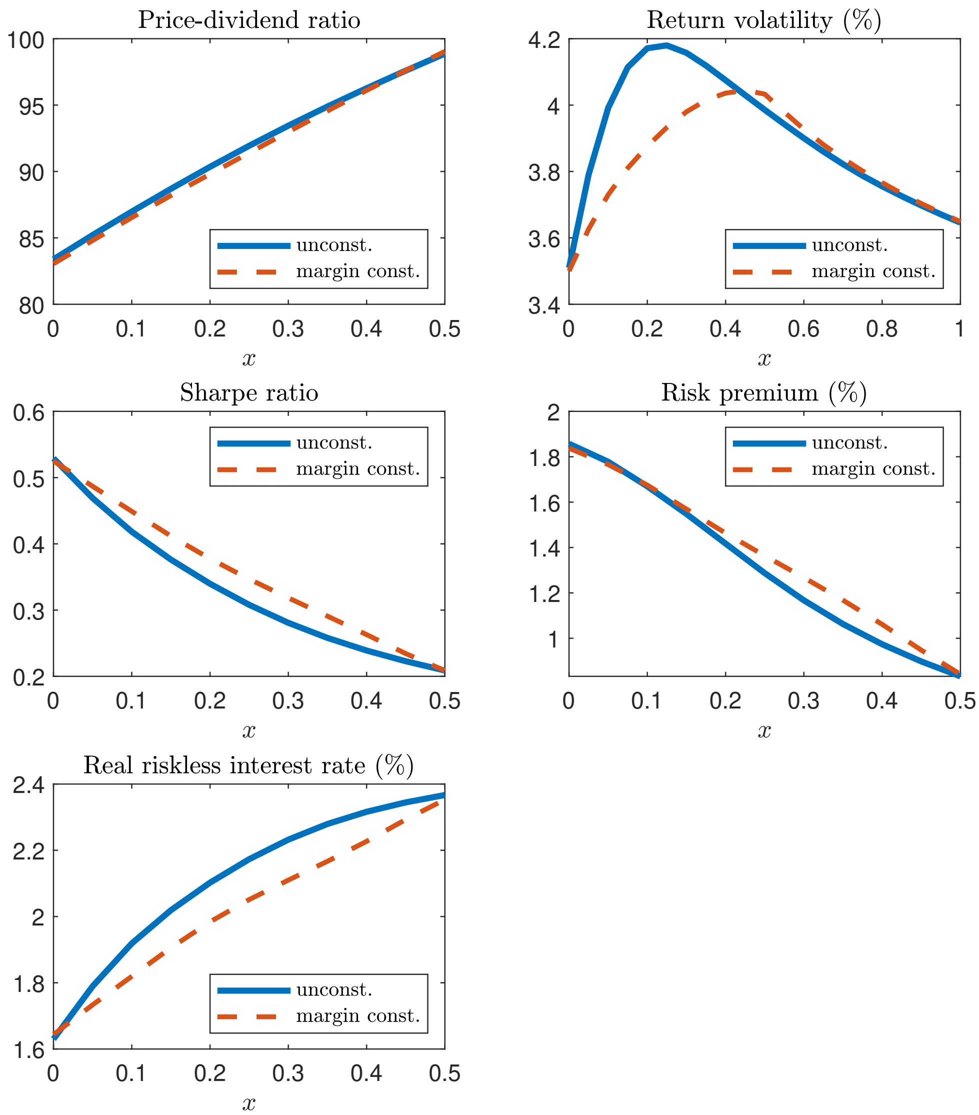

Figure 3: Model Objects with Calibrated Parameters

This graph depicts a selection of objects from the Kargar model that correspond to the author's Figure 2 (p.518). For each variable, y is kept at the value $\bar{y}$.

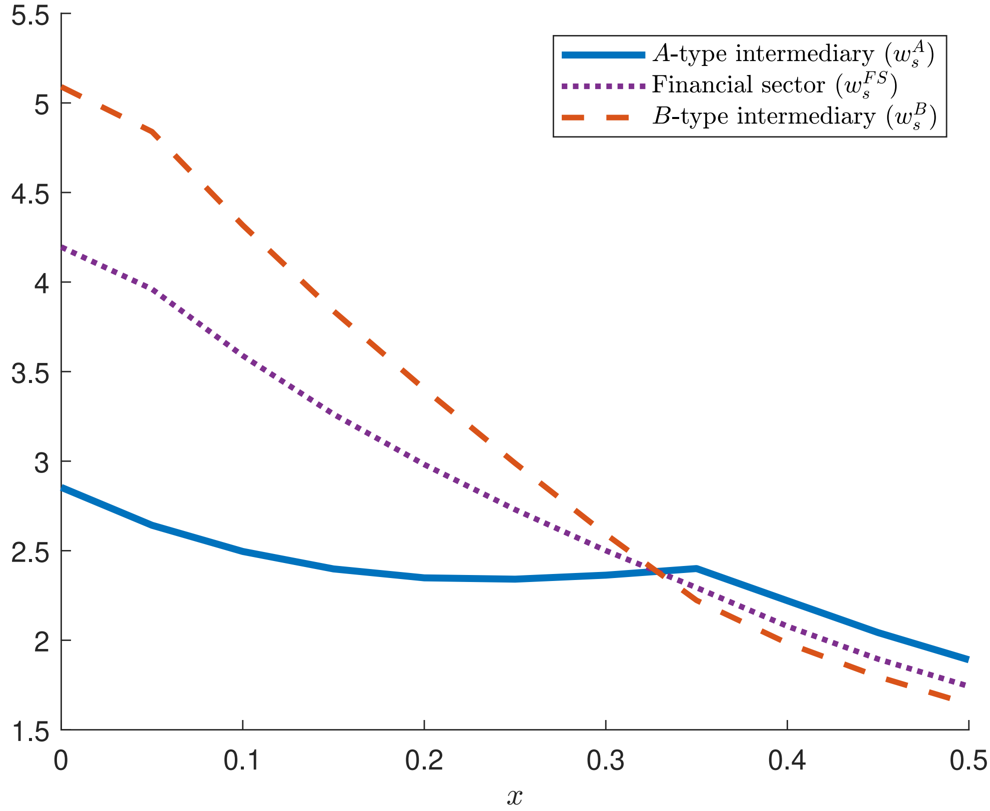

Figure 4: Leverage Plot with Alternative Parameters

This graph depicts the leverage ratios of agents A and B at the alternative parameter values with γ_B being changed to a value of 3. This version roughly coincides with Kargar's Figure 4, although I abstain from smoothing the graphs through interpolation with splines.

responses of different kinds of financial intermediaries to times of market stress, this is the combination of plots that should be presented in the paper accompanied with the correct set of preference parameters that actually allow for an inversion of the leverage order of the model's agents. However, the coupling of results from apparently different parameter sets is a significant mistake and misleads readers of the original paper. Tackling this issue and dealing with, potentially, visually less appealing diffusion plots would be the better course of action.

4 Conclusion

This short comment discusses the DSGE model designed by Kargar (2021). Kargar's paper targets an interesting point of discussion within the asset pricing literature by highlighting the importance of the prevalence of structurally different players in the financial sector. The empirical performance of the additional risk factor is interesting

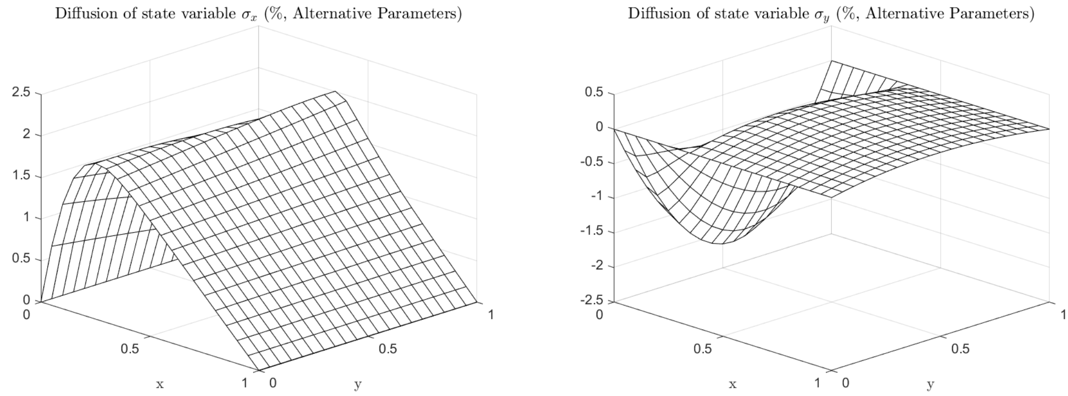

Figure 5: Diffusion Plots with Alternative Parameters

In this figure, the state variable diffusions are again shown for the same alternative parameter set ($\gamma_B' = 3$). However, the significantly negative values for σ_y are in stark contrast to the results communicated by Kargar (2021).

in its own right and seems like a logical next step onwards from Adrian et al. (2014) and He et al. (2017), but the presentation of the theoretical model underlying this empirical application is lacking in several places.

While the model presents an interesting framework of three agents with heterogeneous preferences, a clear reason as to why this is a model of financial intermediaries is missing. Furthermore, the model being calibrated to significantly different levels of the state variables than the empirical sample is at least peculiar. While the introduction of mean reversion terms for the laws of motion of the state variables is ad-hoc and may raise eyebrows for researchers deriving the formulas from scratch, the main issue with this article is the suggestion that the calibrated parameter values actually produce all graphs and figures in the paper. I show that this cannot be the case as two of these graphs contradict each other fundamentally. In this calibration, a key aspect of the model, namely the more risk-averse intermediary B taking on more leverage than A during times of financial distress in the sector (i.e. low values of x) is not fulfilled. However, it is not completely impossible to achieve this dynamic with the model framework, even though state variable diffusion terms will differ significantly in this case with σ_y having to turn negative. Any indication for this is missing from the paper even though it would be crucial information.

A discussion of these issues is necessary for further research on this model, to which I have contributed myself with an estimation study of the preference parameters of the three agents using simulation-based techniques. In that analysis, the parameter estimates also do not generate clearly intersecting leverage paths for the two intermediaries.

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A Correct Numerical Algorithm

This section outlines the numerical algorithm as presented in Kargar's (2021) internet appendix. However, this version includes a crucial system of equations that determines the portfolio weights. While this system is used in Kargar's own code, it does not appear in his documentation, which severely reduces reproducibility of the model results. In this outline of the numerical algorithm, it is discussed in points 2 and 3.

1. Using the goods market clearing condition, we can derive an expression for the dividend yield F and its derivatives with respect to the two state variables. Here, we replace c_i with J_i and use the derivatives of the approximated J_i functions as discussed before:

$$\begin{aligned}
F &= xyJ_A + x(1-y)J_B + (1-x)J_C, \\
F_x &= yJ_A + (1-y)J_B - J_C + xyJ_{A,x} + x(1-y)J_{B,x} + (1-x)J_{C,x}, \\
F_y &= xJ_A - xJ_B + xyJ_{A,y} + x(1-y)J_{B,y} + (1-x)J_{C,y}, \\
F_{xx} &= 2yJ_{A,x} + 2(1-y)J_{B,x} - 2J_{C,x} + xyJ_{A,xx} + x(1-y)J_{B,xx} + (1-x)J_{C,xx}, \\
F_{yy} &= 2xJ_{A,y} - 2xJ_{B,y} + xyJ_{A,yy} + x(1-y)J_{B,yy} + (1-x)J_{C,yy}, \\
F_{xy} &= J_A - J_B + xJ_{A,x} - xJ_{B,x} + yJ_{A,y} + (1-y)J_{B,y} - J_{C,y} + xyJ_{A,xy} \\
&\quad + x(1-y)J_{B,xy} + (1-x)J_{C,xy}.
\end{aligned}$$

2. We then compute portfolio weights for the different agents. Without restrictions, the system of equations (Line 1 holds for agents A, B and C) reads:

$$\begin{aligned}
w_s^i =& \frac{1}{\psi_i}\left[\frac{\mu - r}{\sigma^2} + \frac{1-\gamma_i}{1-\psi_i}\left(\frac{J_{i,x}}{J_i}\frac{\sigma_x}{\sigma} + \frac{J_{i,y}}{J_i}\frac{\sigma_y}{\sigma}\right)\right] \\
1 =& xyw_s^A + x(1-y)w_s^B + (1-x)w_s^C
\end{aligned}$$

Inserting an explicit expression for σ_x/σ and σ_y/σ yields:

$$\begin{aligned}
w_s^i =& \frac{1}{\psi_i}\left[\frac{\mu - r}{\sigma^2} + \frac{1-\gamma_i}{1-\psi_i}\left(\frac{J_{i,x}}{J_i}x\left(yw_s^A + (1-y)w_s^B - 1\right)\right.\right. \\
&\left.\left. + \frac{J_{i,y}}{J_i}y(1-y)\left(w_s^A - w_s^B\right)\right)\right] \\
1 =& xyw_s^A + x(1-y)w_s^B + (1-x)w_s^C
\end{aligned}$$

This yields

$$
\begin{aligned}
\frac{(1-\gamma_A)J_{A,x}}{(1-\psi_A)J_A}x =& \frac{\mu-r}{\sigma^2} + \left[\frac{1-\gamma_A}{1-\psi_A}\left(\frac{J_{A,x}}{J_A}xy + \frac{J_{A,y}}{J_A}y(1-y)\right) - \gamma_A\right] w_A^s \\
&+ \frac{1-\gamma_A}{1-\psi_A}\left(\frac{J_{A,x}}{J_A}x(1-y) - \frac{J_{A,y}}{J_A}y(1-y)\right) w_B^s \\
\frac{(1-\gamma_B)J_{B,x}}{(1-\psi_B)J_B}x =& \frac{\mu-r}{\sigma^2} + \frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}xy + \frac{J_{B,y}}{J_B}y(1-y)\right) w_A^s \\
&+ \left[\frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}x(1-y) - \frac{J_{B,y}}{J_B}y(1-y)\right) - \gamma_B\right] w_B^s \\
\frac{(1-\gamma_C)J_{C,x}}{(1-\psi_C)J_C}x =& \frac{\mu-r}{\sigma^2} + \frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}xy + \frac{J_{C,y}}{J_C}y(1-y)\right) w_A^s \\
&+ \frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}x(1-y) - \frac{J_{C,y}}{J_C}y(1-y)\right) w_B^s - \gamma_C w_C^s \\
1 =& xyw_A^s + x(1-y)w_B^s + (1-x)w_C^s
\end{aligned}
$$

In matrix notation:

$$
\begin{pmatrix} 1 & a_{12} & a_{13} & 0 \\ 1 & a_{22} & a_{23} & 0 \\ 1 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}
\begin{pmatrix} \frac{\mu-r}{\sigma^2} \\ w_A^s \\ w_B^s \\ w_C^s \end{pmatrix}
=
\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{pmatrix}
$$

$$
\Leftrightarrow
\begin{pmatrix} 0 & a_{12}-a_{22} & a_{13}-a_{23} & 0 \\ 1 & a_{22} & a_{23} & 0 \\ 0 & a_{32}-a_{22} & a_{33}-a_{23} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}
\begin{pmatrix} \frac{\mu-r}{\sigma^2} \\ w_A^s \\ w_B^s \\ w_C^s \end{pmatrix}
=
\begin{pmatrix} b_1-b_2 \\ b_2 \\ b_3-b_2 \\ 1 \end{pmatrix}
$$

$$
\Leftrightarrow
\begin{pmatrix} 0 & a_{12}-a_{22} & a_{13}-a_{23} & 0 \\ 1 & a_{22} & a_{23} & 0 \\ 0 & c_1 & c_2 & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}
\begin{pmatrix} \frac{\mu-r}{\sigma^2} \\ w_A^s \\ w_B^s \\ w_C^s \end{pmatrix}
=
\begin{pmatrix} b_1-b_2 \\ b_2 \\ c_3 \\ 1 \end{pmatrix}
$$

where $c_1 = a_{44}(a_{32}-a_{22}) - a_{34}a_{42}$, $c_2 = a_{44}(a_{33}-a_{23}) - a_{34}a_{43}$ and $c_3 = a_{44}(b_3 - b_2) - a_{34}$. The individual matrix elements are

- $a_{12} = \frac{1-\gamma_A}{1-\psi_A}\left(\frac{J_{A,x}}{J_A}xy + \frac{J_{A,y}}{J_A}y(1-y)\right) - \gamma_A$
- $a_{13} = \frac{1-\gamma_A}{1-\psi_A}\left(\frac{J_{A,x}}{J_A}x(1-y) - \frac{J_{A,y}}{J_A}y(1-y)\right)$
- $a_{22} = \frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}xy + \frac{J_{B,y}}{J_B}y(1-y)\right)$
- $a_{23} = \frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}x(1-y) - \frac{J_{B,y}}{J_B}y(1-y)\right) - \gamma_B$

- $a_{32} = \frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}xy + \frac{J_{C,y}}{J_C}y(1-y)\right)$
- $a_{33} = \frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}x(1-y) - \frac{J_{C,y}}{J_C}y(1-y)\right)$
- $a_{34} = -\gamma_C$
- $a_{42} = xy$
- $a_{43} = x(1-y)$
- $a_{44} = 1-x$
- $b_1 = \frac{(1-\gamma_A)J_{A,x}}{(1-\psi_A)J_A}x$
- $b_2 = \frac{(1-\gamma_B)J_{B,x}}{(1-\psi_B)J_B}x$
- $b_3 = \frac{(1-\gamma_C)J_{C,x}}{(1-\psi_C)J_C}x$

This then gives us unrestricted portfolio weights:

$$
\begin{aligned}
w_A^s &= \frac{(b_1-b_2)c_2 - c_3(a_{13}-a_{23})}{(a_{12}-a_{22})c_2 - c_1(a_{13}-a_{23})} \\
w_B^s &= \frac{(a_{12}-a_{22})c_3 - c_1(b_1-b_2)}{(a_{12}-a_{22})c_2 - c_1(a_{13}-a_{23})} \\
w_C^s &= \frac{1}{1-x}(1 - xyw_A^s - x(1-y)w_B^s) \\
\frac{\mu - r}{\sigma^2} &= b_2 - a_{22}w_A^s - a_{23}w_B^s
\end{aligned}
$$

3. If the VaR-Constraint on agent A is binding, we have

$$
\begin{aligned}
w_A^s =& \frac{1}{\alpha\sigma_D}\left(1 + \frac{F_x}{F}x\Big[yw_A^s + (1-y)w_B^s - 1\Big] + \frac{F_y}{F}y(1-y)(w_A^s - w_B^s)\right) \\
w_B^s =& \frac{1}{\psi_B}\left[\frac{\mu-r}{\sigma^2} + \frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}x\left(yw_A^s + (1-y)w_B^s - 1\right)\right.\right. \\
&\left.\left. + \frac{J_{B,y}}{J_B}y(1-y)\left(w_A^s - w_B^s\right)\right)\right] \\
w_C^s =& \frac{1}{\psi_C}\left[\frac{\mu-r}{\sigma^2} + \frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}x\left(yw_A^s + (1-y)w_B^s - 1\right)\right.\right. \\
&\left.\left. + \frac{J_{C,y}}{J_C}y(1-y)\left(w_A^s - w_B^s\right)\right)\right] \\
1 =& xyw_s^A + x(1-y)w_s^B + (1-x)w_s^C
\end{aligned}
$$

This yields

$$\begin{aligned}
\frac{1}{\alpha\sigma_D}\left(\frac{F_x}{F}x-1\right) =&\left[\frac{1}{\alpha\sigma_D}\left(\frac{F_x}{F}xy+\frac{F_y}{F}y(1-y)\right)-1\right]w_A^s \\
&+\frac{1}{\alpha\sigma_D}\left(\frac{F_x}{F}x(1-y)-\frac{F_y}{F}y(1-y)\right)w_B^s \\
\frac{(1-\gamma_B)J_{B,x}}{(1-\psi_B)J_B}x =&\frac{\mu-r}{\sigma^2}+\frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}xy+\frac{J_{B,y}}{J_B}y(1-y)\right)w_A^s \\
&+\left[\frac{1-\gamma_B}{1-\psi_B}\left(\frac{J_{B,x}}{J_B}x(1-y)-\frac{J_{B,y}}{J_B}y(1-y)\right)-\gamma_B\right]w_B^s \\
\frac{(1-\gamma_C)J_{C,x}}{(1-\psi_C)J_C}x =&\frac{\mu-r}{\sigma^2}+\frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}xy+\frac{J_{C,y}}{J_C}y(1-y)\right)w_A^s \\
&+\frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}x(1-y)-\frac{J_{C,y}}{J_C}y(1-y)\right)w_B^s-\gamma_C w_C^s \\
1 =&xyw_A^s+x(1-y)w_B^s+(1-x)w_C^s
\end{aligned}$$

In matrix notation:

$$\begin{aligned}
&\begin{pmatrix} 0 & \tilde{a}_{12} & \tilde{a}_{13} & 0 \\ 1 & a_{22} & a_{23} & 0 \\ 1 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}\begin{pmatrix} \frac{\mu-r}{\sigma^2} \\ w_A^s \\ w_B^s \\ w_C^s \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ b_2 \\ b_3 \\ 1 \end{pmatrix} \\
\Leftrightarrow &\begin{pmatrix} 0 & \tilde{a}_{12} & \tilde{a}_{13} & 0 \\ 1 & a_{22} & a_{23} & 0 \\ 0 & a_{32}-a_{22} & a_{33}-a_{23} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}\begin{pmatrix} \frac{\mu-r}{\sigma^2} \\ w_A^s \\ w_B^s \\ w_C^s \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ b_2 \\ b_3-b_2 \\ 1 \end{pmatrix} \\
\Leftrightarrow &\begin{pmatrix} 0 & \tilde{a}_{12} & \tilde{a}_{13} & 0 \\ 1 & a_{22} & a_{23} & 0 \\ 0 & c_1 & c_2 & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}\begin{pmatrix} \frac{\mu-r}{\sigma^2} \\ w_A^s \\ w_B^s \\ w_C^s \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ b_2 \\ c_3 \\ 1 \end{pmatrix}
\end{aligned}$$

where $c_1 = a_{44}(a_{32}-a_{22}) - a_{34}a_{42}$, $c_2 = a_{44}(a_{33}-a_{23}) - a_{34}a_{43}$ and $c_3 = a_{44}(b_3 - b_2) - a_{34}$. The adjusted matrix elements are

- $\tilde{a}_{12} = \frac{1}{\alpha\sigma_D}\left(\frac{F_x}{F}xy + \frac{F_y}{F}y(1-y)\right) - 1$ and
- $\tilde{a}_{13} = \frac{1}{\alpha\sigma_D}\left(\frac{F_x}{F}x(1-y) - \frac{F_y}{F}y(1-y)\right)$.

This then gives us restricted portfolio weights:

$$
\begin{aligned}
w_A^s &= \frac{\tilde{b}_1 c_2 - c_3(a_{13} - a_{23})}{(a_{12} - a_{22})c_2 - c_1 \tilde{a}_{13}} \\
w_B^s &= \frac{\tilde{a}_{12} c_3 - c_1 \tilde{b}_1}{\tilde{a}_{12} c_2 - c_1 \tilde{a}_{13}} \\
w_C^s &= \frac{1}{1-x}(1 - xyw_A^s - x(1-y)w_B^s) \\
\frac{\mu - r}{\sigma} &= b_2 - a_{22} w_A^s - a_{23} w_B^s
\end{aligned}
$$

4. Given both the restricted and the unrestricted portfolio weights, we compare point wise:

   - If $w_A^{s,unconstrained} \leq w_A^{s,constrained}$, we set $w_i^s = w_i^{s,unconstrained}$

   - If $w_A^{s,unconstrained} > w_A^{s,constrained}$, we set $w_i^s = w_i^{s,constrained}$

5. Since all values are now known, the return volatility can then be computed as:

$$
\sigma = \frac{\sigma_D}{1 + \frac{F_x}{F} x \Big[y w_s^a + (1-y) w_s^B - 1 \Big] + \frac{F_y}{F} y(1-y)(w_s^A - w_s^B)}.
$$

6. Using this volatility, we can then solve for the diffusions of the state variables:

$$
\sigma_x = x\Big[yw_s^A + (1-y)w_s^B - 1\Big]\sigma, \qquad \sigma_y = y(1-y)\Big[w_s^A - w_s^B\Big]\sigma.
$$

7. Using the expression for the return volatility, the state variable diffusions and agent C's portfolio weight, we can compute the expected excess return (the risk premium) on the risky asset as:

$$
\mu - r = \gamma_C w_s^C \sigma^2 - \frac{1-\gamma_C}{1-\psi_C}\left(\frac{J_{C,x}}{J_C}\sigma_x + \frac{J_{C,y}}{J_C}\sigma_y\right)\sigma
$$

8. Using this result, the drifts of the state variables can be computed when substituting J_i for c_i as suggested by the optimal choice of agent i discussed above:

$$
\begin{aligned}
\mu_x &= x\left[\left(yw_s^A + (1-y)w_s^B - 1\right)(\mu - r - \sigma^2) - (yJ_A + (1-y)J_B) + F\right], \\
\mu_y &= y(1-y)\Bigg[(w_s^A - w_s^B)(\mu - r) - J_A + J_B - \left[yw_s^A + (1-y)w_s^B\right]\left(w_s^A - w_s^B\right)\sigma^2\Bigg]
\end{aligned}
$$

9. In the next step, we can find the expected return on the risky asset from:

$$
\begin{aligned}
\mu =& \mu_D + F - \frac{F_x}{F}\left(\kappa(\bar{x} - x) + \mu_x + \sigma_D \sigma_x\right) - \frac{F_y}{F}\left(\kappa(\bar{y} - y) + \mu_y + \sigma_D \sigma_y\right) \\
&+ \left(\left(\frac{F_x}{F}\right)^2 - \frac{1}{2}\frac{F_{xx}}{F}\right)\sigma_x^2 + \left(\left(\frac{F_y}{F}\right)^2 - \frac{1}{2}\frac{F_{yy}}{F}\right)\sigma_y^2 + \left(2\frac{F_x}{F}\frac{F_y}{F} - \frac{F_{xy}}{F}\right)\sigma_x \sigma_y
\end{aligned}
$$

10. Subsequently, the real interest rate in the model must be equal to

$$r = \mu - (\mu - r).$$

11. In a final step, we have to insert the above expressions in the HJB equations:

$$\begin{aligned} 0 = &- (\rho + \kappa) + \frac{1}{\psi_i} J_i + \left(1 - \frac{1}{\psi_i}\right) \left[r + w_s^i(\mu - r) - \frac{\gamma_i}{2} \left(w_s^i\right)^2 \sigma^2\right] \\ &- \frac{1}{\psi_i} \left(\left[\frac{J_{i,x}}{J_i} \left(\kappa(\bar{x} - x) + \mu_x\right) + \frac{J_{i,y}}{J_i} \left(\kappa(\bar{y} - y) + \mu_y\right)\right] \right. \\ &\left. + (1 - \gamma_i) w_s^i \sigma \left[\frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} \sigma_y\right] \right) \\ &- \frac{1}{2\psi_i} \left[\frac{\psi_i - \gamma_i}{1 - \gamma_i} \left(\frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} \sigma_y\right)^2 + \frac{J_{i,xx}}{J_i} \sigma_x^2 + \frac{J_{i,yy}}{J_i} \sigma_y^2 + 2 \frac{J_{i,xy}}{J_i} \sigma_x \sigma_y\right]. \end{aligned}$$

These are the residual functions used for Chebyshev collocation as described in Section 2.1. The aim is to find a set of parameters for the functions J_A, J_B and J_C that sets these residuals to zero for all collocation points chosen in the application. The vector of all $3(N+1)^2 = 3\tilde{N}$ residuals is the objective function of the basic optimization problem.

B Kargar's Figures

This section contains the figures by Kargar (2021) that are discussed in this comment. I include the published graphs in this appendix to facilitate the comparison between the replication results and the officially reported figures.

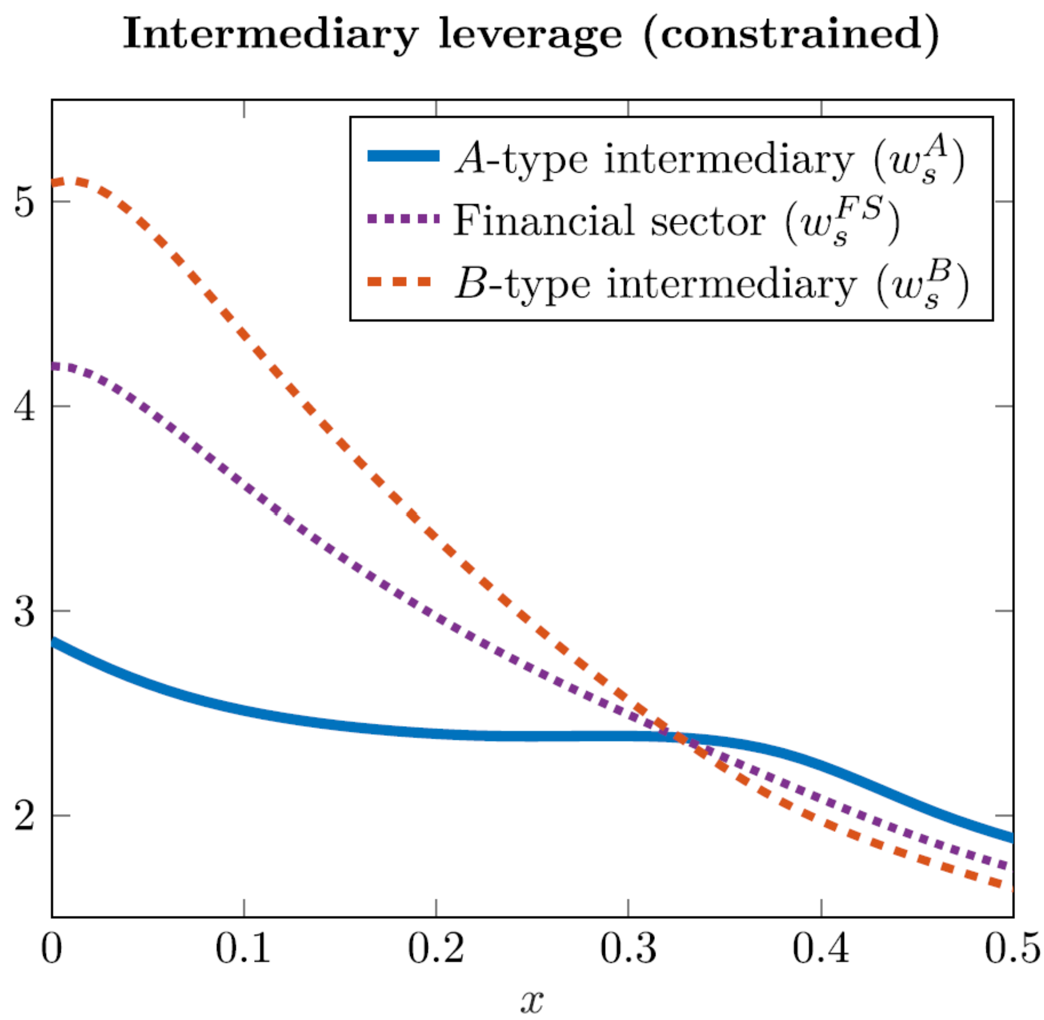

Figure 6: Figure 4 as in Kargar (2021, p.520)

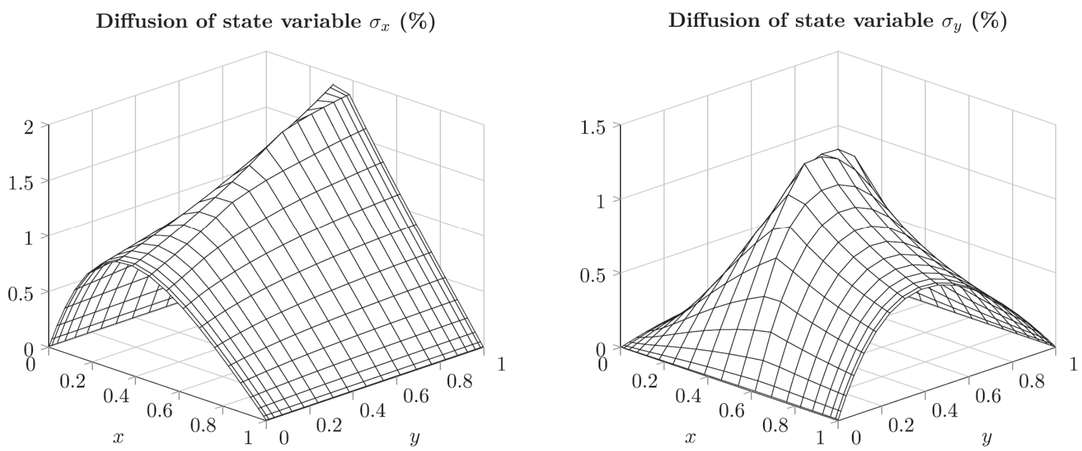

Figure 7: Figure 6 as in Kargar (2021, p.522)

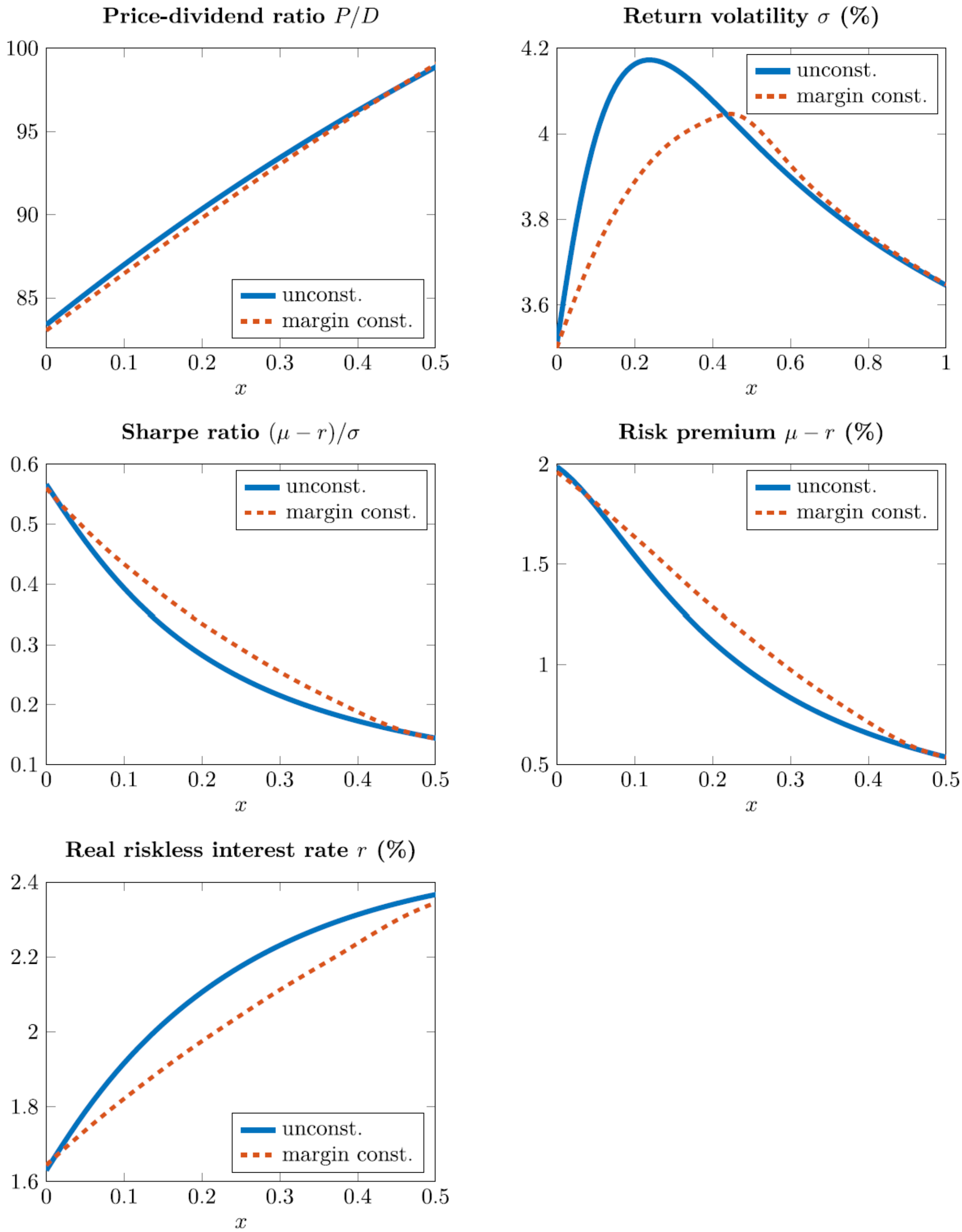

Figure 8: Figure 2 as in Kargar (2021, p.518)